SNR, the rmse($\hat{\boldsymbol{\tau}}$) using only the training sequence is not sufficiently low to yield dependable bit-estimates. In particular, for high values of SNR, where the MAI dominates, the WLS detector is very sensitive to errors in the time-delay estimates. This observation is consistent with results reported in, e.g., [7].

On the other hand, iterating twice and using the $\hat{\mathbf{z}}_1$ matrix to improve the time-delay estimates leads to a substantial decrease in the bit-error probability. This example thus highlights the main idea of the method presented here. Significant improvement in performance is possible by looking at the channel-parameter estimation and symbol detection problems jointly by iterating and exploiting known signal structure.

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Multiple Fully Adaptive Notch Filter Design Based on Allpass Sections

Victor DeBrunner and Sebastian Torres

***Abstract*—We develop a canonical, adaptive cascade-structure IIR notch filter to detect and track multiple time-varying frequencies in additive white Gaussian noise. The algorithm uses allpass frequency transformation filters and a truncated gradient. Simulations indicate that our algorithm is computationally simple, converges rapidly, and has good frequency resolution.**

***Index Terms*—Adaptive filters, allpass filters, notch filters.**

I. INTRODUCTION

The problem of designing adaptive notch filters (ANF's) for retrieving narrowband signals immersed in broadband noise was first introduced in [1], although this ANF did not consider tracking. The constrained ANF in [2] and [3] used a simplified gradient. Further improvement resulted by constraining the zeros of the filter to lie on the unit circle to form sharp notches [4]. Constraining the filter poles to lie on the same radial line as the zeros, but slightly inside the unit circle, was developed independently in [5] and [6]. These ideas were refined in [7]. We estimate both the central frequency and the bandwidth of the notch filter as in [8], resulting in improved tracking capabilities. We also incorporate correction mechanisms for resolving closely spaced sinusoids as well as very low or very high frequencies. Multiple sinusoids are easily incorporated. Our scheme is based on frequency transformations for digital filters; consequently, we call it the frequency-transform based notch filter (FTBNF).

II. FREQUENCY-TRANSFORM BASED NOTCH FILTER

We introduce a new parameterization for an adaptive notch filter by using the frequency transformation of [9] to convert a lowpass notch into a variable-frequency notch. This allows us to preserve spectral features after transformation and retain a low complexity implementation. The adaptation uses a recursive prediction error (RPE) algorithm that incorporates an adaptive time-varying notch bandwidth and forgetting factor. We begin with the analysis for the single-sinusoid case that approximately follows [7].

Consider the first-order notch filter (a lowpass filter structure) given by

$$H_0(z) = \frac{1 + z^{-1}}{1 + \rho z^{-1}}. \tag{1}$$

This filter is a notch filter with center frequency $\omega_0 = \pi$. The filter is limited since the parameter we can modify ρ is related only to the filter bandwidth. Applying the frequency transformation $z^{-1} \rightarrow z^{-1} H_{ap}(z)$, where $H_{ap}(z)$ is a first-order allpass filter yields the filter

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$$H_n(z) = \frac{1 + z^{-1}H_{ap}(z)}{1 + \rho z^{-1}H_{ap}(z)} \equiv \frac{B_n(z)}{A_n(z)} = \frac{1 - 2\alpha z^{-1} + z^{-2}}{1 - (1+\rho)\alpha z^{-1} + \rho z^{-2}}. \tag{2}$$

This filter has zeros on the unit circle at $z_0 = e^{\pm j\omega_0}$, where $\omega_0 = \cos^{-1}\alpha$. For $\rho \to 1$, the poles are located at $z_p \approx \rho e^{\pm j\omega_0}$, thus approximating the constrained notch filter in [5]. The bandwidth is

$$BW = 2\cos^{-1}\left(\frac{2\rho}{1+\rho^2}\right). \tag{3}$$

Adding the constraint

$$\alpha^2 < \frac{4\rho}{(1+\rho)^2} \tag{4}$$

ensures that the filter has complex-conjugate poles while constraining the operating range of the ANF.

We adapt on α and ρ independently. For the FTBNF, we use RPEM. Using the shift operator q^{-1} defined by $q^{-n}[x(t)] = x(t-n)$ with integer n, we can write $A_n(q^{-1})e(t) = B_n(q^{-1})y(t)$, where A_n and B_n retain the parameterization as in (2). Then, using techniques from [10], the gradient is

$$\psi^\alpha(t) \approx \frac{1}{A_n(q^{-1})} q^{-1} \frac{\partial H_{ap}(q^{-1})}{\partial \alpha}[y(t) - \rho e(t)]. \tag{5}$$

The adaptive parameter is α; its derivatives are taken with $\alpha = \alpha(t-1)$. We use the gradient truncation

$$\frac{\partial H_{ap}(q^{-1})}{\partial \alpha} = \frac{\partial}{\partial \alpha}\frac{B_{ap}(q^{-1})}{A_{ap}(q^{-1})} \xrightarrow[\text{truncation}]{} \frac{1}{A_{ap}(q^{-1})}\frac{\partial B_{ap}(q^{-1})}{\partial \alpha} \tag{6}$$

to simplify the gradient as

$$\psi^\alpha(t) \approx \frac{\rho e(t-1) - y(t-1)}{1 - (1+\rho)\alpha q^{-1} + \rho q^{-2}}. \tag{7}$$

The accuracy of the RPE algorithm depends on both the forgetting factor of the estimation algorithm and the pole radius of the notch filter [8], [11]. Consider that as $\rho \to 1$, the filter transients endure longer and the slower the filter can track rapidly changing signals. Therefore, we should vary ρ according to the input signal [8]. As for α, we compute the simplified (and independent) gradient

$$\psi^\rho(t) = \frac{\partial e(t)}{\partial \rho} \approx \frac{\alpha e(t-1) - e(t-2)}{1 - (1+\rho)\alpha q^{-1} + \rho q^{-2}}. \tag{8}$$

We note that the optimal value for ρ and the commonly used forgetting factor λ are equivalent [11], [12]. However, it is not effective to adapt both parameters as one since ρ may vary relatively too quickly, and the algorithm is known to be very sensitive to variations of λ. Thus, we update λ as in [11].

When using the above algorithm, we must ensure the stability of the IIR filter at each step of the recursion. Thus, we constrain the parameters α and ρ to lie inside the stability region of the filter. The stability region is the well-known stability triangle. Unstable poles are projected inside the unit circle, and the convergence is not altered [13].

Cascading several of these sections allows us to handle multiple sinusoids of different frequencies. In this case, the objective is to minimize the output from each individual section. Our RPEM is given in Fig. 2. This extended filter inherits all the properties of the single-sinusoid algorithm:

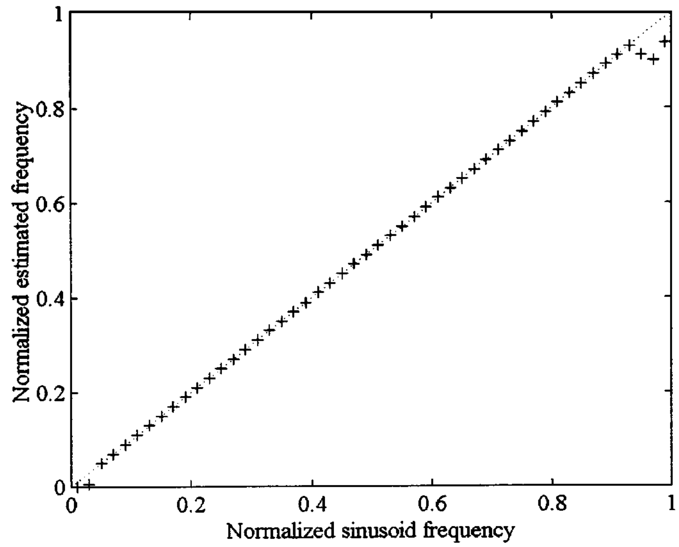

Fig. 1. Estimated frequency versus true frequency for the FTBNF (average of 10 runs).

Parameters: $\mu = 1, \lambda^\rho = 0.99$

Initialization: *For* i=1, …, n

$\alpha_i(0) = 0, \rho_i(0) = \lambda_i^\alpha(0) = 0.7, R_i^\alpha(0) = R_i^\rho(0) = 1$

end *For i*

Computation: *For* t = 1, 2, …, N

$e_0(t) = y(t)$

For i=1, …, n

$e_i(t) = \varepsilon_{i-1}(t) - 2\alpha_i\varepsilon_{i-1}(t-1) + \varepsilon_{i-1}(t-2) + [1+\rho_i(t-1)]\alpha_i(t-1)\varepsilon_i(t-1) - \rho_i(t-1)\varepsilon_i(t-2)$

$\psi_i^\alpha(t) = \nabla_\alpha e_i(t)\big|_{\alpha_i = \alpha_i(t-1), \rho_i = \rho_i(t-1)}$

$\psi_i^\rho(t) = \nabla_\rho e_i(t)\big|_{\alpha_i = \alpha_i(t-1), \rho_i = \rho_i(t-1)}$

$R_i^\alpha(t) = \lambda_i^\alpha R_i^\alpha(t-1) + \psi_i^\alpha(t)^2$

$R_i^\rho(t) = \lambda_i^\rho R_i^\rho(t-1) + \psi_i^\rho(t)^2$

$\alpha_i(t) = \alpha_i(t-1) - \mu\psi_i^\alpha(t)e_i(t)/R_i^\alpha(t)$

$\rho_i(t) = \rho_i(t-1) - \mu\psi_i^\rho(t)e_i(t)/R_i^\rho(t)$

$\varepsilon_i(t) = \varepsilon_{i-1}(t) - 2\alpha_i(t)\varepsilon_{i-1}(t-1) + \varepsilon_{i-1}(t-2) + [1+\rho_i(t)]\alpha_i(t)\varepsilon_i(t-1) - \rho_i(t)\varepsilon_i(t-2)$

$\lambda_i^\alpha(t) = 0.995\lambda_i^\alpha(t-1) + 0.005\rho_i(t-1)$

end *For i*

Stability Projection

Frequency range checking mechanism

Correction mechanism for dealing with close sinusoids

end *For t*

Note: The instantaneous frequency of each signal being tracked is given by $\omega_i(t) = \cos^{-1}\alpha_i(t)$. The output of the filter bank is given by $e_n(t)$.

Fig. 2. Extended RPEM for FTBNF; n is the number of cascaded second-order IIR sections.

- stability monitoring;
- unimodality of the search surface;
- gradient simplicity;
- algorithm structure.

Its performance evaluation is discussed in the next section, where a comparison to the Nehorai's (NANF) and Chamber's (CANF) approaches is performed.

III. Performance Evaluation and Comparison

We have extensively compared our method with two existing schemes [5], [7]. The simulations are programmed in MATLAB™ and run on an IBM PC compatible computer. Because of the brief nature of this correspondence item, only a few results are noted. The general input signal has the form

$$y(t) = \sum_{i=1}^{n} C_i \sin[\pi \varphi_i(t) + \phi_i] + w(t), \quad t = 1, 2, \cdots, N. \tag{9}$$

The noise process $w(t)$ is a zero-mean, white Gaussian process with variance σ^2, ϕ_i are the random initial phases uniformly distributed in the interval $[0, 2\pi)$, and $\varphi_i(t)$ is the normalized phase for the ith sinusoid. For the stationary case, $\varphi_i(t) = f_i t$, and consequently, $f_i(t) = f_i$ is constant. There are several items of concern.

- *Algorithm Convergence*: When choosing ρ, it is important to consider the effect on the notch bandwidth, the shape of the error performance surface, and the range of reachable frequencies

$$\cos^{-1}\left(\frac{2\sqrt{\rho}}{1+\rho}\right) < \omega < \cos^{-1}\left(-\frac{2\sqrt{\rho}}{1+\rho}\right) \tag{10}$$

  obtained by applying the constraint in (4) and using the relationship $\alpha = \cos\omega$. As $\rho \to 1$, the $BW \to 0$, and the reachable range of frequencies becomes the full interval $[0, \pi]$.
- *Estimation Accuracy and Operation Range*: A plot of the true frequency versus the estimated frequency is presented in Fig. 1. Here, the constraint in (4) is included because the convergence to either the very low or the very high frequencies is impeded when using an adaptive ρ. Fortunately, projecting ρ into the region of interest given by (10) alleviates this problem.
- *Filter Tracking Capabilities*: Analysis methods can be found in [14] and [15].

We have learned the following.

- *Stationary Single Sinusoid*: The FTBNF always converges; convergence can be slower than for the NANF or CANF. The NANF and CANF do not converge for very low or very high frequencies.
- *Frequency-Hopping Single Sinusoid*: In the NANF, the evolution of ρ does not depend on the input signal. Consequently, it cannot follow the frequency hop when it occurs too "late." The CANF can follow the hop, but convergence is slower than the FTBNF due to its fixed ρ.
- *Stationary Multiple Sinusoids*: Convergence to the correct frequencies is assured only by implementation of a "corrective mechanism" that employs a switch between an adaptive and a deterministically changing ρ. The NANF does not exhibit good performance, and the CANF will often have several cascaded sections converge to the same sinusoid.
- *Nonstationary Multiple Sinusoids*: We examined the performance for frequency hops, chirps, and polynomial-phase components. The NANF cannot track these (see the single nonstationary sinusoid discussion). The CANF performs very poorly on the FM signals, and its convergence in the case of the frequency hops is slow. The FTBNF tracks sinusoids through frequency crossover.
- *Statistical Analysis*: We repeated the test from [5] for the single stationary sinusoid. Statistically, the NANF is superior, with our FTBNF performing the worst. This result is expected because of the adaptive ρ that allows us to use it for frequency tracking.
- *Computational Complexity*: The NANF has an exponential complexity, whereas the CANF and the FTBNF have a linear complexity, with the FTBNF being slightly more complex than the CANF.

IV. Conclusions

We develop an adaptive notch filter algorithm for tracking nonstationary narrowband signals. We use a frequency transformation network to realize the notch filter that yields a low-complexity gradient computation. We have a simple stability check, even when used for multiple narrowband components.

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