STAGGERED PRT WITH GROUND CLUTTER FILTERING AND OVERLAID ECHO RECOVERY FOR DUAL POLARIZATION

ALGORITHM DESCRIPTION

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PREFACE

This document extends the previous Staggered PRT algorithm description from July 2009 by including dual polarization sequences and the calculation of polarimetric variables. This algorithm description includes a high-level description with the overall processing logic followed by a detailed explanation of each pre-computation and processing step.

Most of the July 2009 algorithm steps are now repeated for both H- and V-channels calculation. Unlike them, the SACHI filter has been modified preserving the phase value in order to allow the calculation of the polarimetric variables. Now, the output of the SACHI filter produces autocorrelations for both H- and V-channels and also the cross-correlation between them. To ease implementation and reduce ambiguity, most of the steps in the SACHI algorithm are described in algorithmic form. The DC removal ground clutter filter has been retained to operate on those range gates where only long-PRT data is available and ground clutter filtering is needed and now it includes both channels.

As in the July 2009 description, the algorithm is able to handle overlaid echoes, extending the recovery of Doppler moments to the unambiguous range of the long PRT. Moment-specific overlaid power thresholds are used to identify recoverable data and flag unrecoverable Doppler moments. In this version of the algorithm, ground clutter is also assumed to be within the unambiguous range of the short PRT.

1) The transmission sequence alternates two pulse repetition times (PRT) as: $T_1, T_2, T_1, T_2 \ldots$ for a total of M pulses.

2) The PRT ratio $T_1/T_2 = 2/3$, where $\kappa_m = 2$, $\kappa_n = 3$ and $T_2 - T_1 = T_u$.

3) All range gates are available and there is a perfect alignment of range gates between the two PRTs (i.e., a given range gate represents the same resolution volume in space for every transmitted pulse). Also, the number of range gates for each PRT is: $N_1 = T_1/\tau_s$ and $N_2 = T_2/\tau_s$, where τ_s is the sampling period.

4) There are no significant echoes beyond the maximum unambiguous range corresponding to T_2 (r_{a2}).

5) There is no significant ground clutter beyond the maximum unambiguous range corresponding to T_1 (r_{a1}).

6) The number of staggered PRT samples per range gate (M) is even.

7) The algorithm operates on a radial worth of data at a time.

INPUTS

1) Dual polarization complex time-series data:

$$\begin{cases} V_H(n, m) = I_H(n, m) + jQ_H(n, m), \\ V_V(n, m) = I_V(n, m) + jQ_V(n, m), \end{cases}$$

where subscripts H and V denote horizontal and vertical polarization, $0 \leq n < N_1$ for even m, $0 \leq n < N_2$ for odd m and $0 \leq m < M$. Note that n indexes the range gates and m the sweeps (or pulses).

2) Associated metadata:

λ is the radar wavelength in meters
N_H is the noise power in linear units for the horizontal channel
N_V is the noise power in linear units for the vertical channel
$dBZ0$ is the system calibration constant in dB
$ATMOS$ is the elevation-dependent atmospheric attenuation in dB/km
ΔR is the spacing between range gates in km ($\Delta R = c\tau_s/2$)
T_Z is the signal-to-noise ratio threshold for reflectivity in dB
T_V is the signal-to-noise ratio threshold for velocity in dB
T_W is the signal-to-noise ratio threshold for spectrum width in dB
T_{OV} is the velocity overlaid threshold in dB (*Note: recommended value is 0 dB*)
T_{OW} is the spectrum width overlaid threshold in dB (*Note: recommended value is 10 dB*)

3) Data window:

$d'(m)$, where $0 \leq m < 5M/2$. Note that d' does not need to be normalized or scaled in any way. A tapered data window such as the Blackman window is recommended for best performance of the SACHI ground clutter filter. Otherwise, rectangular window (i.e., no window) should be applied.

4) Ground clutter filter bypass map:

$B(n)$, where n indexes the range bins with the same resolution as the time-series data along a radial, and the map corresponds to the elevation and azimuth of the radial being processed. B is 0 if clutter filtering is required and 1 otherwise. In this algorithm, the clutter map is ignored beyond the unambiguous range corresponding to the short PRT where clutter is assumed not to be present.

OUTPUTS

1) Reflectivity, Doppler velocity, and spectrum width calculated from H channel data:

$Z(n)$ for $0 \leq n < N_2$,
$v(n)$ and $w(n)$ for $0 \leq n < N_2$.

2) Differential reflectivity, differential phase and correlation coefficient calculated from H and V channel data:

$Z_{DR}(n)$ for $0 \leq n < N_2$,
$\Phi_{DP}(n)$ for $0 \leq n < N_2$,
$\rho_{HV}(n)$ for $0 \leq n < N_2$.

3) Signal-to-noise ratio and overlaid censoring flags*:

$NS_Z(n)$, $NS_V(n)$ and $NS_W(n)$ for $0 \leq n < N_2$,
$OV_V(n)$ and $OV_W(n)$ for $0 \leq n < N_2$.

* $NS_Z(n)$ is used for censoring $Z_{DR}(n)$, $\Phi_{DP}(n)$ and $\rho_{HV}(n)$.

FUNCTIONS AND CONVENTIONS

1) $|\cdot|$ – Returns the absolute value of a complex number or the absolute value of each element of a matrix of complex numbers.

2) arg – Returns the principal phase angle of the input complex number in radians. The algorithm is written to accommodate this phase in the interval $[0, 2\pi)$ or $[-\pi, \pi)$.

3) $\underset{k}{\arg\min}$ – Returns the index k to the element in the input vector that has the minimum value.

4) diag – Returns a square matrix with the input vector along the principal diagonal (row index = column index) of the matrix and all other elements not on the principal diagonal equal to zero. The number of rows (columns) of the matrix is equal to the number of elements in the vector.

5) ceiling – Returns the smallest integer value not less than the input number.

6) floor – Returns the largest integer value not greater than the input number.

7) round – Returns the nearest integer to the input number.

8) max – Returns the maximum value among the input numbers.

9) Italicized names are used to denote scalars (e.g., *Noise*).

10) Bolded names are used to denote vectors or matrices (e.g., **A**). Italicized names with indexing in parentheses are used to denote elements of a vector or matrix [e.g., $A(i,j)$].

11) * – Denotes complex conjugate.

12) T – Denotes matrix transpose.

13) j – Denotes the imaginary unit $\sqrt{-1}$.

If first run of SPRT algorithm

    1) Pre-computation of velocity dealiasing rules

    2) Pre-computation of M-independent SACHI filter parameters

End

If the number of samples (M) changed

    3) Pre-computation of window parameters

    4) Pre-computation of M-dependent SACHI filter parameters

End

For each range bin n, where $0 \leq n < N_2$

    If $n \geq N_1$

        5) Short-PRT Segment-III Data Reconstruction

    End

    If $B(n) = 0$ AND $n < N_1$

        6) SACHI Clutter Filtering (Segment-I/II gate with segment-I/II clutter)

    Else

        If $n \geq N_1$ AND $B(n - N_1) = 0$

            7) DC Removal Clutter Filtering (Segment-III gate with segment-I clutter)

        Else

            8) No Clutter Filtering

        End

        9) Power and correlation computations for each PRT

        10) Combined power and cross-correlation computation

    End

End

11) Strong point clutter canceling

For each range bin n, where $0 \leq n < N_2$

    12) Signal power computation

    13) Reflectivity computation

    14) Velocity computation

    15) Spectrum width computation

    16) Differential reflectivity computation

    17) Differential phase computation

    18) Cross-correlation coefficient computation

    19) Determination of significant returns for reflectivity and polarimetric variables

    20) Determination of significant returns for velocity

    21) Determination of significant returns for spectrum width

End

For each range bin n, where $0 \leq n < N_2$

    22) Determination of overlaid returns for velocity and spectrum width

End

1) Pre-computation of velocity dealiasing rules

This method is described in the paper "Design, Implementation, and Demonstration of a Staggered PRT Algorithm for the WSR-88D" by Torres et al. (2004). Herein, VDA_c are the normalized velocity difference transfer function (VDTF) constant values and VDA_p are the normalized number of Nyquist co-intervals for dealiasing.

A set of velocity dealiasing rules is pre-computed at the initiation of the SPRT algorithm as follows:

(Compute type-I and II positive VDTF discontinuity points. κ_m and κ_n are the integers in the PRT ratio)
$p = 0$
While $2p + 1 < \kappa_m$
    $D_1(p) = (2p + 1)/\kappa_m$
    $TYPE_1(p) = 1$
    $p = p + 1$
End
$q = 0$
While $2q + 1 < \kappa_n$
    $D_2(q) = (2q + 1)/\kappa_n$
    $TYPE_2(q) = 2$
    $q = q + 1$
End

(Create TYPE by combining and sorting both sets of discontinuity points)
Concatenate D_1 and D_2 to create D with $p + q$ elements.
Concatenate $TYPE_1$ and $TYPE_2$ to create $TYPE$ with $p + q$ elements.
Sort $TYPE$ in a "slave" mode using D as the "master".

(Compute VDTF constants and dealiasing factors for non-negative discontinuity points)
$VDA_c(p + q) = 0$
$VDA_p(p + q) = 0$
For $0 \leq k < p + q$
    If $TYPE(k) = 1$
        $VDA_c(p + q + k + 1) = VDA_c(p + q + k) - 2/\kappa_m$
        $VDA_p(p + q + k + 1) = VDA_p(p + q + k) + 1/\kappa_m$
    Else
        $VDA_c(p + q + k + 1) = VDA_c(p + q + k) + 2/\kappa_n$
        $VDA_p(p + q + k + 1) = VDA_p(p + q + k)$
    End
End

(Compute VDTF constants and dealiasing factors for negative discontinuity points)
For $-(p + q) \leq k < 0$
    $VDA_c(p + q + k) = -VDA_c(p + q - k)$
    $VDA_p(p + q + k) = -VDA_p(p + q - k)$
End

(Note that since the PRT ratio does not change, these vectors can be hard-coded in a real-time implementation of the SPRT algorithm.)

2) Pre-computation of *M*-independent SACHI filter parameters

This method is described in NSSL Signal Design and Processing Techniques for WSR-88D Ambiguity Resolution (Report 3, Report 9 and Report 11). The SACHI filter parameters could be pre-computed at the initiation of the SPRT algorithm as follows:

(Create 5-by-5 convolution matrix, $\mathbf{C_r}$)

$$\mathbf{C_r} = \begin{bmatrix} C(0) & C(4) & C(3) & C(2) & C(1) \\ C(1) & C(0) & C(4) & C(3) & C(2) \\ C(2) & C(1) & C(0) & C(4) & C(3) \\ C(3) & C(2) & C(1) & C(0) & C(4) \\ C(4) & C(3) & C(2) & C(1) & C(0) \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{r,1} & \mathbf{C}_{r,2} & \mathbf{C}_{r,3} & \mathbf{C}_{r,4} & \mathbf{C}_{r,5} \end{bmatrix},$$

where $C(k) = \dfrac{1}{\sqrt{10}}\sum\limits_{n=0}^{4} c(n)\exp(-j2\pi nk/5)$; for $0 \le k < 5$ and $\mathbf{c} = [1, 0, 1, 0, 0]$, and $\mathbf{C}_{r,k}$ is the *k*-th <u>column</u> of $\mathbf{C_r}$.

(Calculate magnitude deconvolution matrix, $\mathbf{C_{md}}$)
(Note: The following formulas are written in matrix algebra notation with the conventions described above)

$$\mathbf{C_{md}} = |\mathbf{C_r}|^{-1} = \begin{bmatrix} \mathbf{C}_{md,1} \\ \mathbf{C}_{md,2} \\ \mathbf{C}_{md,3} \\ \mathbf{C}_{md,4} \\ \mathbf{C}_{md,5} \end{bmatrix} = \begin{bmatrix} -4.6281 & -2.0697 & 4.6281 & 4.6281 & -2.0697 \\ -2.0697 & -4.6281 & -2.0697 & 4.6281 & 4.6281 \\ 4.6281 & -2.0697 & -4.6281 & -2.0697 & 4.6281 \\ 4.6281 & 4.6281 & -2.0697 & -4.6281 & -2.0697 \\ -2.0697 & 4.6281 & 4.6281 & -2.0697 & -4.6281 \end{bmatrix},$$

where $\mathbf{C}_{md,k}$ is the *k*-th <u>row</u> of $\mathbf{C_{md}}$.

(Calculate matrices $\mathbf{C_{f1}}$ and $\mathbf{C_{f2}}$ using 1st and 5th columns of $\mathbf{C_r}$)

$$\mathbf{C_{f1}} = \mathbf{C}_{r,1}\mathbf{C}_{r,1}^{*T} = \begin{bmatrix} 0.4 & 0.0382+j0.1176 & 0.2618-j0.1902 & 0.2618+j0.1902 & 0.0382-j0.1176 \\ 0.0382-j0.1176 & 0.0382 & -0.0309-j0.0951 & 0.0809-j0.0588 & -0.0309-j0.0225 \\ 0.2618+j0.1902 & -0.0309+j0.0951 & 0.2681 & 0.0809+j0.2490 & 0.0809-j0.0588 \\ 0.2618-j0.1902 & 0.0809+j0.0588 & 0.0809-j0.2490 & 0.2618 & -0.0309-j0.0951 \\ 0.0382+j0.1176 & -0.0309+j0.0225 & 0.0809+j0.0588 & -0.0309+j0.0951 & 0.0382 \end{bmatrix}$$

$$\mathbf{C_{f2}} = \mathbf{C}_{r,5}\mathbf{C}_{r,5}^{*T} = \begin{bmatrix} 0.0382 & -0.0309-j0.0951 & 0.0809-j0.0588 & -0.0309-j0.0225 & 0.0382-j0.1176 \\ -0.0309+j0.0951 & 0.2618 & 0.0809+j0.249 & 0.0809-j0.0588 & 0.2618+j0.1902 \\ 0.0809+j0.0588 & 0.0809-j0.249 & 0.2618 & -0.0309-j0.0951 & 0.2618-j0.1902 \\ -0.0309+j0.0225 & 0.0809+j0.0588 & -0.0309+j0.0951 & 0.0382 & 0.0382+j0.1176 \\ 0.0382+j0.1176 & 0.2618-j0.1902 & 0.2618+j0.1902 & 0.0382-j0.1176 & 0.4 \end{bmatrix}$$

where $*T$ stands for the matrix conjugate transpose (a.k.a. Hermitian) operation.

(Calculate the correction coefficients ξ_2 and ξ_3 for correction vector $\mathbf{X}$)

$$\xi_k = \frac{1}{\mathbf{C}_{md,1}\left|\mathbf{C}_{r,k}-\left(\mathbf{C}_{r,1}^{*T}\mathbf{C}_{r,k}\right)\mathbf{C}_{r,1}\right|}; k = 2, 3.$$

$\xi_2 = 1.1056$ and $\xi_3 = 1.7889$.

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(Note: since the PRT ratio does not change, these matrices and coefficients can be hard-coded in a real-time implementation of the SPRT algorithm. The numbers provided here are for reference purposes only; the highest precision available is recommended for hard-coding these numbers.)

3) Pre-computation of window parameters

(Calculate the extended number of coefficients)
$M_x = 5M / 2$

(Calculate the number of pulse pairs)
$M_p = M / 2$

(Calculate normalized window d for un-normalized window function d' with M_x points)

$$d(m) = d'(m)\left(\sqrt{\frac{1}{M_x}\sum_{m=0}^{M_x-1}[d'(m)]^2}\right)^{-1} ; \quad 0 \le m < M_x.$$

(Calculate window correction factor for lag-1)

$$d_c = \frac{1}{M_x}\sum_{m=0}^{M_x-2} d(m)d(m+1)$$

4) Pre-computation of *M*-dependent SACHI filter parameters

*(Compute correction vector, **X**)*
For $0 \le k < \text{ceiling}(M_p/2)$
    $X(k) = 1$
End
For $\text{ceiling}(M_p/2) \le k < \text{ceiling}(M_p/2) + M_p$
    $X(k) = \xi_2$
End
For $\text{ceiling}(M_p/2) + M_p \le k < \text{ceiling}(M_p/2) + 3M_p$
    $X(k) = \xi_3$
End
For $\text{ceiling}(M_p/2) + 3M_p \le k < \text{ceiling}(M_p/2) + 4M_p$
    $X(k) = \xi_2$
End
For $\text{ceiling}(M_p/2) + 4M_p \le k < M_x$
    $X(k) = 1$
End

5) Short-PRT Segment-III Data Reconstruction

Long-PRT Segment-I data is used as a proxy for short-PRT segment-III data

For $0 \leq m < M_p$
$\quad V_H(n, 2m) = V_H(n - N_1, 2m + 1)$
$\quad V_V(n, 2m) = V_V(n - N_1, 2m + 1)$
End

6) SACHI Clutter Filtering

The SACHI filter algorithm is used when clutter filtering is required inside the maximum unambiguous range corresponding to T_1 (r_{a1}).

(Form derived time series, V_{Hd} and V_{Vd}, from input time series V_H and V_V)
For $0 \leq m < M_p$
$\quad V_{Hd}(5m) = V_H(n, 2m)$
$\quad V_{Hd}(5m + 1) = 0$
$\quad V_{Hd}(5m + 2) = V_H(n, 2m + 1)$
$\quad V_{Hd}(5m + 3) = 0$
$\quad V_{Hd}(5m + 4) = 0$

$\quad V_{Vd}(5m) = V_V(n, 2m)$
$\quad V_{Vd}(5m + 1) = 0$
$\quad V_{Vd}(5m + 2) = V_V(n, 2m + 1)$
$\quad V_{Vd}(5m + 3) = 0$
$\quad V_{Vd}(5m + 4) = 0$

End

(Compute DFT of windowed extended time series power compensated for added zeroes)

$$F_H(k) = \left(\sqrt{\frac{5}{2}}\right)\left(\frac{1}{M_x}\sum_{m=0}^{M_x-1} V_{Hd}(m)d(m)\exp(-j2\pi km/M_x)\right); k = 0, 1 \ldots, M_x - 1.$$

$$F_V(k) = \left(\sqrt{\frac{5}{2}}\right)\left(\frac{1}{M_x}\sum_{m=0}^{M_x-1} V_{Vd}(m)d(m)\exp(-j2\pi km/M_x)\right); k = 0, 1 \ldots, M_x - 1.$$

(Determine clutter filter width parameter, q)
(Use GMAP to return the number of coefficients identified as clutter, $GMAP_{Hcoef}$ and $GMAP_{Vcoeff}$. Pass to GMAP the 5th of the Doppler spectrum containing the main clutter replica; i.e., $|F_{\{H,V\}}(0)|^2,..., |F_{\{H,V\}}[\text{ceiling}(M_p/2) - 1]|^2$, $|F_{\{H,V\}}[M_x - \text{floor}(M_p/2)]|^2,..., |F_{\{H,V\}}(M_x - 1)|^2\}$; initialize GMAP for spectra with $v_a/5$, and get the number of coefficients identified as clutter to estimate q for both H and V channels)

$q_H = floor\ [(GMAP_{Hcoef} + 1)/2]$
$q_V = floor\ [(GMAP_{Vcoef} + 1)/2]$

(Use the largest q to create the clutter filter vectors for polarimetric variable calculation)

$q' = max(q_H, q_V)$

(Create clutter filter vectors I_{f1}', I_{f2}')

For $0 \leq k < M_p$
 If $k < q$'
  I_{f1}' $(k) = 1$
  I_{f2}' $(k) = 0$

 ElseIf $k \leq M_p - q$'
  I_{f1}' $(k) = 0$
  I_{f2}' $(k) = 0$
 Else
  I_{f1}' $(k) = 0$
  I_{f2}' $(k) = 1$
 End
End

(Row-wise re-arrange F_H and F_V into 5-by-M_p matrices, $\mathbf{F_{Hr}}$ and $\mathbf{F_{Vr}}$)
 For $0 \leq k < M_p$
  $F_{Hr}(0, k) = F_H(k)$
  $F_{Hr}(1, k) = F_H(k + M_p)$
  $F_{Hr}(2, k) = F_H(k + 2M_p)$
  $F_{Hr}(3, k) = F_H(k + 3M_p)$
  $F_{Hr}(4, k) = F_H(k + 4M_p)$

  $F_{Vr}(0, k) = F_V(k)$
  $F_{Vr}(1, k) = F_V(k + M_p)$
  $F_{Vr}(2, k) = F_V(k + 2M_p)$
  $F_{Vr}(3, k) = F_V(k + 3M_p)$
  $F_{Vr}(4, k) = F_V(k + 4M_p)$
 End

(Compute the clutter filtered spectrum matrices, $\mathbf{F_{Hf}}$ and $\mathbf{F_{Vf}}$)
(Note: The following formulas are written in matrix algebra notation. Complex-matrix multiplications can be implemented using four real-matrix multiplications as: $\mathbf{AB} = (\mathbf{A_r} + j\mathbf{A_i})(\mathbf{B_r} + j\mathbf{B_i}) = (\mathbf{A_r B_r} - \mathbf{A_i B_i}) + j(\mathbf{A_r B_i} + \mathbf{A_i B_r})$)

$$\mathbf{F_{Hf}} = \mathbf{F_{Hr}} - \mathbf{C_{f1} F_{Hr}} \, \text{diag}\left(\mathbf{I_{f1}}'\right) - \mathbf{C_{f2} F_{Hr}} \, \text{diag}\left(\mathbf{I_{f2}}'\right)$$

$$\mathbf{F_{Vf}} = \mathbf{F_{Vr}} - \mathbf{C_{f1} F_{Vr}} \, \text{diag}\left(\mathbf{I_{f1}}'\right) - \mathbf{C_{f2} F_{Vr}} \, \text{diag}\left(\mathbf{I_{f2}}'\right)$$

(Row-wise unfold $\mathbf{F_{Hf}}$ and $\mathbf{F_{Vf}}$ into F_{Hdf} and F_{Vdf})
For $0 \leq k < M_p$
 $F_{Hdf}(k) = F_{Hf}(0, k)$
 $F_{Hdf}(k + M_p) = F_{Hf}(1, k)$
 $F_{Hdf}(k + 2M_p) = F_{Hf}(2, k)$
 $F_{Hdf}(k + 3M_p) = F_{Hf}(3, k)$
 $F_{Hdf}(k + 4M_p) = F_{Hf}(4, k)$

 $F_{Vdf}(k) = F_{Vf}(0, k)$
 $F_{Vdf}(k + M_p) = F_{Vf}(1, k)$

$$F_{Vdf}(k + 2M_p) = F_{Vf}(2, k)$$
$$F_{Vdf}(k + 3M_p) = F_{Vf}(3, k)$$
$$F_{Vdf}(k + 4M_p) = F_{Vf}(4, k)$$
End

(Compute mean power for both channels, P_H' and P_V, and cross-correlation at lag 0, $R_{HV}(0)$)

$$P'_H(n) = \sum_{k=0}^{M_x-1} \left| F_{Hdf}(k) \right|^2$$

$$P_V(n) = \sum_{k=0}^{M_x-1} \left| F_{Vdf}(k) \right|^2$$

$$R_{HV}(n) = \sum_{k=0}^{M_x-1} F^*_{Hdf}(k) F_{Vdf}(k)$$

(Proceed with the conventional SACHI Clutter Filtering, only H-channel data)
$q = q_H$

If $q < q'$

    (Create clutter filter vectors I_{f1}, I_{f2})
    For $0 \le k < M_p$
        If $k < q$
            $I_{f1}(k) = 1$
            $I_{f2}(k) = 0$

        ElseIf $k \le M_p - q$
            $I_{f1}(k) = 0$
            $I_{f2}(k) = 0$
        Else
            $I_{f1}(k) = 0$
            $I_{f2}(k) = 1$
        End
    End

$$\mathbf{F_f} = \mathbf{F_{Hr}} - \mathbf{C_{f1}}\mathbf{F_{Hr}} \, \mathrm{diag}\left(\mathbf{I_{f1}}\right) - \mathbf{C_{f2}}\mathbf{F_{Hr}} \, \mathrm{diag}\left(\mathbf{I_{f2}}\right)$$

Else

$$\mathbf{F_f} = \mathbf{F_{Hf}}$$

End

(Create clutter filter vectors, $\mathbf{I_1}$, and $\mathbf{I_2}$)
For $0 \le k < M_p$
    If $k < q$
        $I_1(k) = 0$
        $I_1(k + M_p) = 0$
        $I_1(k + 2M_p) = 0$

$$I_1(k + 3M_p) = 0$$
$$I_1(k + 4M_p) = 0$$
$$I_2(k) = 1$$
$$I_2(k + M_p) = 1$$
$$I_2(k + 2M_p) = 1$$
$$I_2(k + 3M_p) = 1$$
$$I_2(k + 4M_p) = 1$$

ElseIf $k \le M_p - q$

$$I_1(k) = 1$$
$$I_1(k + M_p) = 1$$
$$I_1(k + 2M_p) = 1$$
$$I_1(k + 3M_p) = 1$$
$$I_1(k + 4M_p) = 1$$
$$I_2(k) = 0$$
$$I_2(k + M_p) = 0$$
$$I_2(k + 2M_p) = 0$$
$$I_2(k + 3M_p) = 0$$
$$I_2(k + 4M_p) = 0$$

Else

$$I_1(k) = 0$$
$$I_1(k + M_p) = 0$$
$$I_1(k + 2M_p) = 0$$
$$I_1(k + 3M_p) = 0$$
$$I_1(k + 4M_p) = 0$$
$$I_2(k) = 1$$
$$I_2(k + M_p) = 1$$
$$I_2(k + 2M_p) = 1$$
$$I_2(k + 3M_p) = 1$$
$$I_2(k + 4M_p) = 1$$

End

End

(Magnitude deconvolved matrix, $\mathbf{F_d}$)

$$\mathbf{F_d} = \mathbf{C_{md}} \left| \mathbf{F_f} \right|$$

(Row-wise unfold $\mathbf{F_d}$ into F_{df})

For $0 \le k < M_p$

$$F_{df}(k) = F_d(0, k)$$
$$F_{df}(k + M_p) = F_d(1, k)$$
$$F_{df}(k + 2M_p) = F_d(2, k)$$
$$F_{df}(k + 3M_p) = F_d(3, k)$$
$$F_{df}(k + 4M_p) = F_d(4, k)$$

End

(Compute the lag-1 autocorrelation, R_{1df})

$$R_{1df} = \frac{1}{d_c} \sum_{k=0}^{M_x - 1} \left| F_{df}(k) \right|^2 \exp\left(j2\pi k / M_x \right)$$

(Compute vector $\mathbf{I_v}$ with M/2 ones centered on $\arg(R_{1df})$)

(Round to the nearest spectral coefficient. Choose symmetric window of coefficients around it)

$$k_{0df} = \text{round}\left[\frac{M_x \arg\left(R_{1df}\right)}{2\pi}\right]$$

If $k_{0df} < 0$

    $k_{0df} = k_{0df} + M_x$

End

If $k_{0df} \geq M_x$

    $k_{0df} = k_{0df} - M_x$

End

$$k_{1df} = k_{0df} - \text{floor}(M/4)$$

If $k_{1df} < 0$

    $k_{1df} = k_{1df} + M_x$

End

$$k_{2df} = k_{0df} + \text{ceiling}(M/4) - 1$$

If $k_{2df} \geq M_x$

    $k_{2df} = k_{2df} - M_x$

End

(k_{0df} is the coefficient corresponding to $\arg(R_{1df})$, k_{1df} and k_{2df} specify the extent of M_p spectral coefficients centered on the mean velocity. If $k_{1df} < k_{2df}$, the ones span from k_{1df} to k_{2df}; otherwise, the ones will span from k_{1df} to $M_x - 1$, and 0 to k_{2df})

If $k_{1df} < k_{2df}$

    For $0 \leq k < M_x$

        If $k < k_{1df}$ OR $k > k_{2df}$

            $I_v(k) = 0$

        Else

            $I_v(k) = 1$

        End

    End

Else

    For $0 \leq k < M_x$

        If $k < k_{1df}$ AND $k > k_{2df}$

            $I_v(k) = 0$

        Else

            $I_v(k) = 1$

        End

    End

End

(Interpolate the elements for the region around zero velocity in F_{df} with linearly interpolated values from S_1 and S_2)

If $q > 0$

$$S_1 = \left|F_{df}(q)\right|^2$$

$$S_2 = \left|F_{df}(M_x - q)\right|^2$$

    For $0 \leq k < M_x$

        If $k < q$

$$F_i(k) = \left[S_2 + \left(S_1 - S_2\right)\left(q + k\right)/2q\right]^{1/2}$$

        ElseIf $k > M_x - q$

$$F_i(k) = \left[S_2 + \left(S_1 - S_2\right)\left(q + k - M_x\right)/2q\right]^{1/2}$$

        Else

$$F_i(k) = F_{df}(k)$$
        End
    End
Else
    (Don't interpolate if not needed)
    For $0 \le k < M_x$
$$F_i(k) = F_{df}(k)$$
    End
End

(Compute the corrected spectrum, F_c)
For $0 \le k < M_x$
    $F_c(k) = F_i(k)\, I_1(k) + F_i(k)\, I_2(k)\, I_v(k)\, X(k)$
End

(Compute vector $\mathbf{I_c}$ with ones where there's a non-zero spectral component in vector $\mathbf{F_c}$)
For $0 \le k < M_x$
    $I_c(k) = I_1(k) + I_2(k)\, I_v(k)$
End

(Compute the mean power, P_c, and autocorrelation at lag T_u, R_{1c}, using F_c)
$$P_c = \sum_{k=0}^{M_x-1} \left| F_c(k) \right|^2$$

$$R_{1c} = \frac{1}{d_c} \sum_{k=0}^{M_x-1} \left| F_c(k) \right|^2 \exp\left(j 2\pi k / M_x \right)$$

(Retain only M coefficients centered on velocity based on R_{1c} and delete the rest from F_c and I_c)
$$k_{0c} = \mathrm{round}\left[\frac{M_x \arg\left(R_{1c} \right)}{2\pi} \right]$$

If $k_{0c} < 0$
    $k_{0c} = k_{0c} + M_x$
End
If $k_{0c} \ge M_x$
    $k_{0c} = k_{0c} - M_x$
End
$k_{1c} = k_{0c} - M_p$
If $k_{1c} < 0$
    $k_{1c} = k_{1c} + M_x$
End
$k_{2c} = k_{0c} + M_p - 1$
If $k_{2c} \ge M_x$
    $k_{2c} = k_{2c} - M_x$
End

If $k_{1c} < k_{2c}$
    For $0 \le k < M_x$
        If $k < k_{1c}$ OR $k > k_{2c}$
            $F_m(k) = 0$
            $I_m(k) = 0$
        Else

$$F_m(k) = F_c(k)$$
$$I_m(k) = I_c(k)$$
      End
    End
Else
    For $0 \le k < M_x$
        If $k < k_{1c}$ AND $k > k_{2c}$
$$F_m(k) = 0$$
$$I_m(k) = 0$$
        Else
$$F_m(k) = F_c(k)$$
$$I_m(k) = I_c(k)$$
        End
    End
End

(Compute the modified mean power, P_m, and autocorrelation at lag T_u, R_{1m}, using F_m)

$$P_m = \sum_{k=0}^{M_x-1} |F_m(k)|^2$$

$$R_{1m} = \frac{1}{d_c} \sum_{k=0}^{M_x-1} |F_m(k)|^2 \exp\left(j2\pi k / M_x\right)$$

(Compute noise correction factors)

$$N_c = \frac{1}{M_x} \sum_{k=0}^{M_x-1} I_c(k)$$

$$N_m = \frac{1}{M_x} \sum_{k=0}^{M_x-1} I_m(k)$$

(Compute overlaid power correction if in segment I)
If $n < N_2 - N_1$

$$S_{ov} = \frac{1}{2}\left[\frac{1}{M_p} \sum_{m=0}^{M_p-1} \left|V_H\left(n + N_1, 2m+1\right)\right|^2 - Noise \right]$$

    If $S_{ov} < 0$
$$S_{ov} = 0$$
    End
Else
    $S_{ov} = 0$
End

(Correct powers to remove overlaid contamination adjusted for each spectrum)
$P_m = P_m - N_m S_{ov}$
If $P_m < 0$

    $P_m = 0$

End
$P_c = P_c - N_c S_{ov}$
If $P_c < 0$

    $P_c = 0$

End

(Compute spectrum width power ratio adjustment)

$S_m = P_m - N_m \, Noise$

If $S_m < 0$

$\qquad S_m = 0$

End

If $S_m > 0$

$$P_{adj} = \frac{|R_{1m}|}{S_m}$$

Else

$\qquad P_{adj} = 0$

End

(Compute signal power)

$S_c = P_c - N_c \, Noise$

If $S_c < 0$

$\qquad S_c = 0$

End

(Compute short PRT autocorrelation at lag T_1)

$$R_{H1}(n) = S_c \cdot P_{adj}^4 \exp\left[j2\arg\left(R_{1c}\right) \right]$$

(Compute long PRT autocorrelation at lag T_2)

$$R_{H2}(n) = S_H \cdot P_{adj}^9 \exp\left[j3\arg\left(R_{1c}\right) \right]$$

(Adjust signal power to include noise)

$P_H(n) = S_c + Noise$

(Note that the outputs of SACHI are $P'_H(n)$, $P_V(n)$, $R_{HV}(n)$, $P_H(n)$, $R_{H1}(n)$ and $R_{H2}(n)$)

---

7) DC Removal Clutter Filtering (Segment-III gate with segment-I clutter)

This DC Removal clutter filtering algorithm removes the mean (DC) component of the short-PRT segment-III gates in those locations where the site-dependent clutter filter bypass map B indicates the need for clutter within segment I.

(Calculate the mean of the even pulses.)

$$V_{Hm} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_H(n, 2m)$$

$$V_{Vm} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_V(n, 2m)$$

(Subtract mean from even pulses.)

For $0 \leq m < M_p$

$\qquad V_{HF}(2m) = V_H(n, 2m) - V_{Hm}$

$\qquad V_{HF}(2m + 1) = V_H(n, 2m + 1)$

$\qquad V_{VF}(2m) = V_V(n, 2m) - V_{Vm}$

$\qquad V_{VF}(2m + 1) = V_V(n, 2m + 1)$

End

---

8) No Clutter Filtering

For $0 \le m < M$
$$V_{HF}(m) = V_H(n, m)$$
$$V_{VF}(m) = V_V(n, m)$$

End

---

9) Power and correlation computations for each PRT

If $n < N_1$

(Compute power from even pulses, if available)

$$P_{H1} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} |V_{HF}(2m)|^2$$

$$P_{V1} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} |V_{VF}(2m)|^2$$

(Compute cross-correlation from even pulses, if available)

$$R_{HV1}(n) = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_{HF}^*(2m) V_{VF}(2m)$$

End

(Compute power from odd pulses)

$$P_{H2} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} |V_{HF}(2m+1)|^2$$

$$P_{V2} = \frac{1}{M_p} \sum_{m=0}^{M_p-1} |V_{VF}(2m+1)|^2$$

(Compute cross-correlation from odd pulses)

$$R_{HV2}(n) = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_{HF}^*(2m+1) V_{VF}(2m+1)$$

(Compute lag-1 correlations from all pulses from H channel)

$$R_{H1}(n) = \frac{1}{M_p} \sum_{m=0}^{M_p-1} V_{HF}^*(2m) V_{HF}(2m+1)$$

$$R_{H2}(n) = \frac{1}{M_p-1} \sum_{m=0}^{M_p-2} V_{HF}^*(2m+1) V_{HF}(2m+2)$$

---

10) Combined power and cross-correlation computation

To use as much information as possible, data are extracted from the two power arrays with different rules for each of the three segments depicted in Figure 1. For segment I, data are extracted only from P_1, since P_2 may be contaminated on those range bins with overlaid powers. An average of P_1 and P_2 is extracted for segment II, given that both power vectors are "clean" there. Finally, segment III data are obtained from P_2. In algorithmic form:

If $n < N_2 - N_1$
    (Segment I)
    $P_H(n) = P_{H1}$
    $P_V(n) = P_{V1}$
ElseIf $n < N_1$
    (Segment II)
    $P_H(n) = \frac{1}{2}(P_{H1} + P_{H2})$
    $P_V(n) = \frac{1}{2}(P_{V1} + P_{V2})$
Else
    (Segment III)
    $P_H(n) = P_{H2}$
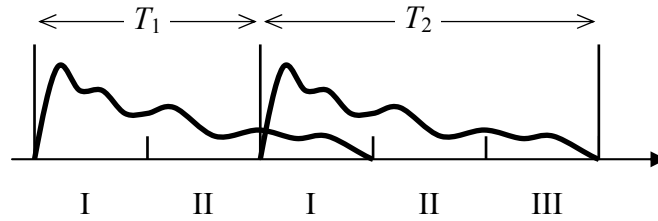 $P_V(n) = P_{V2}$
End
$P'_H(n) = P_H$

Fig. 1. Signal powers in the staggered PRT algorithm. Roman numerals indicate segment numbers.

The same rules apply for the cross-correlation computation.

If $n < N_2 - N_1$
    (Segment I)
    $R_{HV}(n) = R_{HV1}$
ElseIf $n < N_1$
    (Segment II)
    $R_{HV}(n) = \frac{1}{2}(R_{HV1} + R_{HV2})$
Else
    (Segment III)
    $R_{HV}(n) = R_{HV2}$
End

11) Strong point clutter canceling

Processing is as in the current system. Strong-point clutter canceling is applied to P_H, P'_H, R_{H1} and R_{H2} based on radial power continuity in P_H. For the remainder of the algorithm it is assumed that the outputs of this step are P_H, P'_H, R_{H1} and R_{H2}.

12) Signal power computation

If $P_H(n) < N_H$
    $S_H = 0$
Else
    $S_H = P_H(n) - N_H$
End

If $P'_H(n) < N_H$

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$S'_H = 0$

Else

$S'_H = P_H'(n) - N_H$

End

If $P_V(n) < N_V$

$S_V = 0$

Else

$S_V = P_V(n) - N_V$

End

13) Reflectivity computation

(Range in km)
$R = n\Delta R + \Delta R/2$

(Reflectivity in dBZ. $\log_{10}$ *is the base-10 logarithm)*
If $S_H > 0$

$Z(n) = 10\log_{10}(S_H) + dBZ0 + R\ ATMOS + 20\log_{10}(R) - 10\log_{10}(N_H),$

Else

$Z(n)$ should be set to the smallest possible reflectivity value

End

14) Velocity computation

(Compute Doppler velocities for each PRT using the corresponding correlation estimates)

$$v_1 = -\frac{\lambda}{4\pi T_1}\arg\left[R_{H1}(n)\right]$$

$$v_2 = -\frac{\lambda}{4\pi T_2}\arg\left[R_{H2}(n)\right]$$

(Compute extended Nyquist velocity)

$$v_a = \frac{\lambda}{2T_1}$$

(Dealias velocity using pre-computed rules)

$$l = \arg\min_k\left|v_1 - v_2 - VDA_c(k)v_a\right|$$

$$v(n) = v_1 + 2v_a\ VDA_p(l)$$

(Prevent dealiased velocities outside of the extended Nyquist co-interval)
If $v(n) > v_a$

$v(n) = v(n) - 2v_a$

End
If $v(n) < -v_a$

$v(n) = v(n) + 2v_a$

End

15) Spectrum width computation

The spectrum width estimator corresponds to the algorithm implemented in the legacy WSR-88D signal processor.

If $S_H = 0$ OR $|R_{H1}(n)| = 0$

    (Insert spectrum width of white noise)

$$w(n) = \frac{\lambda}{4\sqrt{3}T_1}$$

ElseIf $S_H < |R_{H1}(n)|$

    (Insert spectrum width of a constant)
    $w(n) = 0$

Else

    (Spectrum width computation. ln *is the natural logarithm)*

$$w(n) = \frac{\lambda}{2\sqrt{2}\pi T_1} \sqrt{\ln\left(\frac{S}{|R_{H1}(n)|}\right)}$$

    If $w(n) > \dfrac{\lambda}{4\sqrt{3}T_1}$

        $w(n) = \dfrac{\lambda}{4\sqrt{3}T_1}$

    End

End

16) Differential reflectivity computation

If $S'_H > 0$ AND $S_V > 0$

$$Z_{DR}(n) = 10 \log_{10} \frac{S'_H}{S_V}$$

ElseIf $S'_H = 0$

    $Z_{DR}(n)$ should be set to the smallest possible value

ElseIf $S_V = 0$

    $Z_{DR}(n)$ should be set to the highest possible value

End

17) Differential phase computation

$$\Phi_{DP}(n) = \arg\left[R_{HV}(n)\right]$$

18) Cross-correlation coefficient computation

If $S'_H > 0$ AND $S_V > 0$

$$\rho_{HV}(n) = \frac{|R_{HV}(n)|}{\sqrt{S'_H S_V}}$$

Else

$$\rho_{HV}(n) = 0$$

End

19) Determination of significant returns for reflectivity and polarimetric variables

The non-significant return indicator array (NS_Z) is a binary array where 0 indicates "significant" and 1 indicates "non-significant". This array is also used for $Z_{DR}(n)$, $\Phi_{DP}(n)$ and $\rho_{hv}(n)$.

If $S_H < N_H \cdot 10^{0.1T_Z}$
    $NS_Z(n) = 1$
Else
    $NS_Z(n) = 0$
End

20) Determination of significant returns for velocity

The non-significant return indicator array (NS_V) is a binary array where 0 indicates "significant" and 1 indicates "non-significant"

If $S_H < N_H \cdot 10^{0.1T_V}$
    $NS_V(n) = 1$
Else
    $NS_V(n) = 0$
End

21) Determination of significant returns for spectrum width

The non-significant return indicator array (NS_W) is a binary array where 0 indicates "significant" and 1 indicates "non-significant"

If $S_H < N_H \cdot 10^{0.1T_W}$
    $NS_W(n) = 1$
Else
    $NS_W(n) = 0$
End

22) Determination of overlaid returns for velocity and spectrum width

Censoring of velocity and spectrum width data is only necessary in segments I and III. This is done by analyzing P in segment I (P_1) and P in segment III (P_2) (see Fig. 1). The idea is to determine whether second trip signals mask first trip signals and vice versa. While such overlaid echoes appear in every other pulse and do not bias velocity estimates at those range locations, overlaid powers act as noise. Therefore, when overlaid powers are above a preset fraction of their non-overlaid counterparts, the corresponding velocity and spectrum width estimates exhibit very large errors and must be censored. The overlaid indicator arrays (OV_V and OV_W) are binary arrays where 0 indicates "not overlaid" and 1 indicates "overlaid".

If $n < N_2 - N_1$
    (Segment I: Range gates that may or may not have overlaid echoes)
    (Check power ratio using velocity threshold)

If $P_H(n) > P_H(n + N_1)$ $10^{0.1T_{OV}}$
    $OV_V(n) = 0$
Else
    (Power ratio not met, but consider non-significant returns as non-existent)
    If $NS_V(n + N_1) = 1$
        $OV_V(n) = 0$
    Else
        $OV_V(n) = 1$
    End
End
(Check power ratio using width threshold)
If $P_H(n) > P_H(n + N_1)$ $10^{0.1T_{OW}}$
    $OV_W(n) = 0$
Else
    (Power ratio not met, but consider non-significant returns as non-existent)
    If $NS_W(n + N_1) = 1$
        $OV_W(n) = 0$
    Else
        $OV_W(n) = 1$
    End
End
ElseIf $n < N_1$
    (Segment II: Range gates that, based on the assumptions, never have overlaid echoes)
    $OV_V(n) = 0$
    $OV_W(n) = 0$
Else
    (Segment III: Range gates that may or may not have overlaid echoes)
    (Check power ratio using velocity threshold)
    If $P_H(n) > P_H(n - N_1)$ $10^{0.1T_{OV}}$
        $OV_V(n) = 0$
    Else
        (Power ratio not met, but consider non-significant returns as non-existent)
        If $NS_V(n - N_1) = 1$
            $OV_V(n) = 0$
        Else
            $OV_V(n) = 1$
        End
    End
    (Check power ratio using width threshold)
    If $P_H(n) > P_H(n - N_1)$ $10^{0.1T_{OW}}$
        $OV_W(n) = 0$
    Else
        (Power ratio not met, but consider non-significant returns as non-existent)
        If $NS_W(n - N_1) = 1$
            $OV_W(n) = 0$
        Else
            $OV_W(n) = 1$
        End
    End
End

(Note that when processing the overlaid and significant return flags, the overlaid flags take a lower priority. That is, if a range bin is tagged as non significant and also as overlaid, the overlaid indication is ignored and the gate is treated as a non-significant return only; e.g., painted black as opposed to purple)