P7.8 ANALYSIS OF THE NEW NEXRAD SPECTRUM WIDTH ESTIMATOR

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1. INTRODUCTION

Recently, the US network of weather surveillance radars (NEXRAD) was upgraded with the Open Radar Data Acquisition (ORDA) subsystem which includes new receivers, signal processors, and control subsystems (Patel and Macemon 2004). Before this upgrade, the legacy RDA estimated the spectrum width using the standard pulse-pair technique. The new signal processor implements a similar spectrum width estimator, but relies on a Discrete Fourier Transform (DFT)-based estimator to compute the first few lags of the time-series autocorrelation function. Initial evaluation of the upgraded system demonstrated that, if combined with a tapered data window, the DFT-based estimator produces results that are acceptable but exhibit about 30% larger errors compared to the legacy RDA. This paper demonstrates that, in general, the new and legacy autocorrelation estimators are not equivalent, resulting in inconsistent spectrum width estimates. Theoretical, simulation, and data analyses show that the new spectrum width estimator on non-windowed data is positively biased, especially for narrow spectrum widths. Given that biased estimates would negatively impact the performance of algorithms that rely on the spectrum width (e.g., the radar echo classifier, or the new turbulence detection algorithm), we propose changes to the new spectrum width estimator to make it unbiased, mathematically equivalent to the pulse-pair implementation, and naturally able to handle data window effects.

2. SPECTRUM WIDTH ESTIMATION

The spectrum width σ_v is the square root of the second central moment of the Doppler spectrum and is a measure of velocity dispersion (e.g., shear and turbulence) in the resolution volume. Spectrum width can be used to aid in the interpretation of weather data, leading to improved warnings of severe weather and hazards to aviation (Lemon 1999, Cornman et al. 1999).

The US Weather Surveillance Radar 1988 Doppler (WSR-88D) routinely measures the spectrum width for each resolution volume assuming that the weather spectra have Gaussian shape. With this parameterization, the autocorrelation function of weather signals takes the following form

$$R(mT_s) = Se^{-8(\pi\sigma_v mT_s/\lambda)^2} e^{-j4\pi\bar{v}mT_s/\lambda} + N\delta(mT_s)\,, \qquad (1)$$

where S and N are the signal and noise powers, λ is the radar wavelength, T_s is the pulse repetition time, m is the autocorrelation lag, and $\bar{v}$ is the mean Doppler velocity (Doviak and Zrnić 1993).

From (1), it is possible to show that the spectrum width can be computed from the magnitude of the ratio of autocorrelation values for two different lags. One of the most commonly used spectrum width estimators is the one based on the ratio of estimates of the lag-zero and lag-one autocorrelations (Zrnić 1979):

$$\hat{\sigma}_v = \frac{\lambda}{2\sqrt{2}\pi T_s}\left|\ln\left(\frac{\hat{R}(0)-N}{\left|\hat{R}(1)\right|}\right)\right|^{1/2}. \qquad (2)$$

It is well known that the performance of this estimator deteriorates for low signal-to-noise ratios (SNR) and for either very narrow or very large spectrum widths (Doviak and Zrnić 1993). In spite of its limitations, both the legacy RDA and the ORDA adopted the spectrum width estimator in (2). However, unlike the legacy RDA which only used the rectangular data window, the ORDA spectrum width estimator includes empirical adjustments for broadening in the case of using a tapered data windowing:

$$\sigma_v^{\dagger} = \sqrt{\sigma_v^2 - \sigma_d^2}\,. \qquad (3)$$

In this equation, σ_d is the empirically-determined spectrum width of any of the available tapered data windows (i.e., Hamming, von Hann, or Blackman) and there is no adjustment for rectangular window (σ_d = 0).

As expected, the performance of the spectrum width estimator is tied to that of the autocorrelation estimator; biased autocorrelation estimates lead to biased spectrum widths. In what follows, we explore two approaches to estimating the autocorrelation function for weather signals. The time-domain approach is usually termed as pulse-pair processing and is the scheme employed in the legacy RDA. A DFT-based approach is currently used in the ORDA (Passarelli and Siggia 1983).

2.1. *Time-domain Autocorrelation Estimation*

In the time-domain approach, the lag-zero and lag-one autocorrelation values are estimated from the M samples comprising the time-series data for one resolution volume as

$$\hat{R}_{PPP}(0) = \frac{1}{M}\sum_{m=0}^{M-1}\left|V(m)\right|^2\text{, and} \qquad (4)$$

$$\hat{R}_{PPP}(1) = \frac{1}{M-1}\sum_{m=0}^{M-2}V^*(m)V(m+1)\,. \qquad (5)$$

Herein, the subscript PPP is used to denote pulse-pair processing. Note that the estimator in (5) is based on

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the classical unbiased sample autocorrelation estimator and does not include data windowing. If using a data window, the autocorrelation estimator can be generalized as

$$\hat{R}_{PPP}(l) = \frac{\sum_{m=0}^{M-l-1} V^*(m)V(m+l)}{\sum_{m=0}^{M-l-1} d(m)d(m+l)}; l = 0,1, \tag{6}$$

where d is the data window applied to the time series data before the autocorrelation estimation is carried out. Note that (6) reduces to (4) and (5) for l = 0 and 1 (lags zero and one) for a rectangular data window. However, unlike (4) and (5), the estimator in (6) is unbiased for any data window, not just for a rectangular data window.

2.2. *DFT-based Autocorrelation Estimation*

ORDA performs clutter filtering in the spectral domain, so it makes sense that it uses a DFT-based autocorrelation estimator. The Doppler spectrum for each range gate is estimated from the windowed time-series data as

$$\hat{S}(k) = \left| \frac{1}{M} \sum_{m=0}^{M-1} V(m) e^{-j\frac{2\pi mk}{M}} \right|^2, k = 0,...,M-1, \tag{7}$$

from which the autocorrelation is computed as

$$\hat{R}_{DFT}(l) = \sum_{k=0}^{M-1} \hat{S}(k) e^{j\frac{2\pi kl}{M}}; l = 0,1. \tag{8}$$

Here, the subscript DFT denotes the DFT-based estimator. Substituting (7) into (8) and after simple mathematical manipulations we get

$$\begin{aligned} \hat{R}_{DFT}(l) = \frac{1}{M} \sum_{m=0}^{M-l-1} V^*(m)V(m+l) + \\ + \frac{1}{M} \sum_{m=M-l}^{M-1} V^*(m)V(m+l-M). \end{aligned} \tag{9}$$

Note that this is equivalent to performing a circular correlation on V. Whereas the first term of this equation is analogous to the pulse-pair formulation in which pairs are spaced by l, the second term involves non-coherent pairs spaced by $l–M$. Clearly, these spurious terms are a source of error for the autocorrelation estimator. The expected value of (9) is

$$\begin{aligned} E\left[\hat{R}_{DFT}(l)\right] = R(l)\left[\frac{1}{M} \sum_{m=0}^{M-l-1} d(m)d(m+l)\right] + \\ + R(l-M)\left[\frac{1}{M} \sum_{m=M-l}^{M-1} d(m)d(m+l-M)\right]. \end{aligned} \tag{10}$$

Therefore, the autocorrelation estimator given in (8) is biased due to a multiplicative factor that accounts for data windowing effects and spurious terms involving non-coherent pairs. Note that the only case in which the spurious terms become coherent is if the time-series data is periodic and a whole number of periods fit perfectly in the dwell time (M samples). Obviously, this is seldom the case for weather signals. However, if a tapered data window is used, the spurious terms are negligible since they involve products of samples at either end of the time series.

If a tapered data window is used, the ORDA spectrum width is not affected by either the spurious terms or the improper scaling in the autocorrelation estimator (there is an adjustment due to data windowing broadening). However, for a rectangular window, end samples are not weighed any differently and there is no adjustment to the spectrum width. Therefore, the ORDA spectrum width estimator can be significantly biased if using a rectangular data window.

3. PERFORMANCE OF THE ORDA SPECTRUM WIDTH ESTIMATOR

The ORDA spectrum width estimator described before was shown to be biased, especially if using a rectangular data window for which the DFT-based autocorrelation estimator is biased the most. This spectrum width bias can be easily observed by processing recorded time-series data using the rectangular and Hamming data windows. Figures 1 and 2 show the reflectivity and spectrum width fields for data collected on September 11, 2006 with the National Severe Storms Laboratory's KOUN radar in Norman, OK. Data was recorded and then processed off-line with an ORDA system. Echoes to the south and southwest exhibit significant contamination by anomalous propagation (AP) clutter. Clearly, the spectrum width in this region is expected to be very narrow. Note however that the smaller spectrum width values observed with the Hamming window are contrasted by the higher (biased) values observed with the rectangular window. Compared to the Hamming window, the bias from the rectangular window data can be as high as 3.5 m/s. Algorithms such as the Radar Echo Classifier (REC) which rely on the spectrum width to identify AP clutter may produce erroneous classification results if a rectangular window is used to process the data. Hence, it is imperative to modify the ORDA spectrum width estimator if a rectangular window will be used to process time series data.

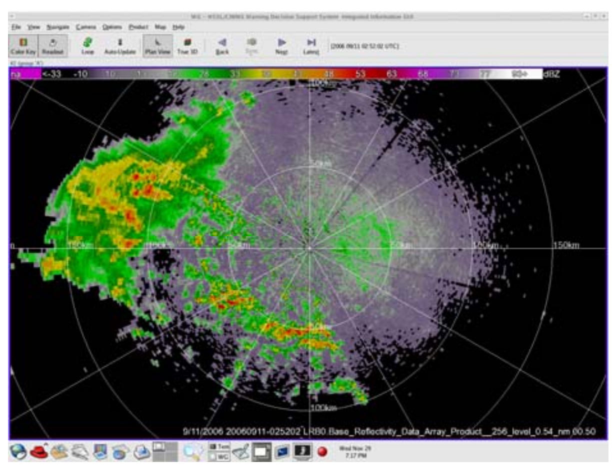

Fig. 1. Reflectivity field from time-series data collected with the KOUN radar on 09/11/06.

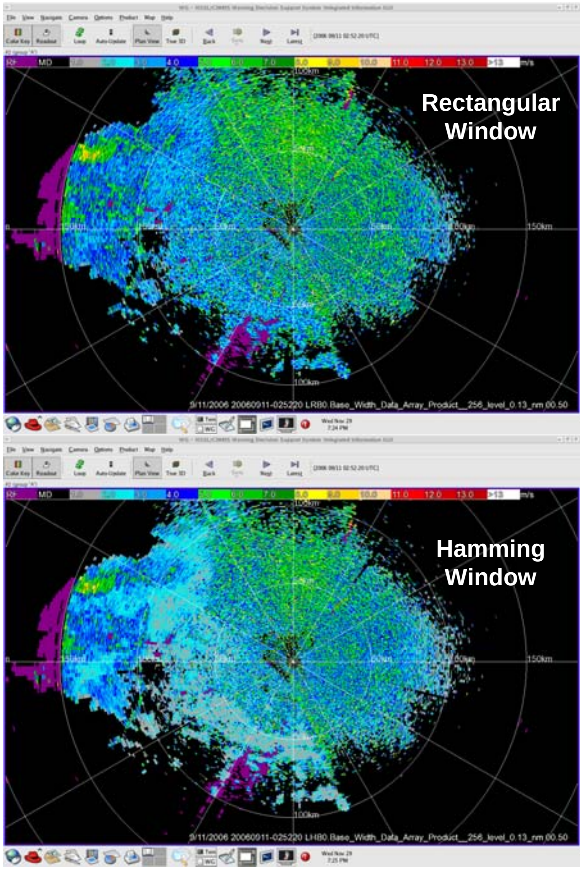

Fig. 2. Spectrum width fields from time-series data collected with the KOUN radar on 09/11/06 and processed with the rectangular window (top) and the Hamming window (bottom).

Simulations were carried out to study the ORDA spectrum width bias in a more systematic manner. Time-series data with varying parameters were simulated using the well-known method by Zrnić (1975). Fig. 3 shows the ORDA spectrum width estimator bias in m/s as a function of the true spectrum width and the mean Doppler velocity of the weather signal for data processed with a rectangular window. In general, as the spectrum width increases, the spectrum width bias decreases uniformly regardless of the mean Doppler velocity. On the other hand, for very narrow spectrum widths there is a dependency of the bias on the mean Doppler velocity. The spectrum width bias is smallest for those specific frequencies (or Doppler velocities) that are multiples of $(MT_S)^{-1}$(i.e., an entire number of periods fit exactly in the dwell time) for which the spurious terms contribute coherently to the estimation process. However, for other frequencies (or Doppler velocities) the spectrum width bias can be as high as 2.5 m/s.

Theoretical, simulation, and data analyses have shown a significant bias in the ORDA spectrum width estimator if time-series data is processed with the rectangular data window. We propose a modified DFT-based autocorrelation estimator to mitigate this problem.

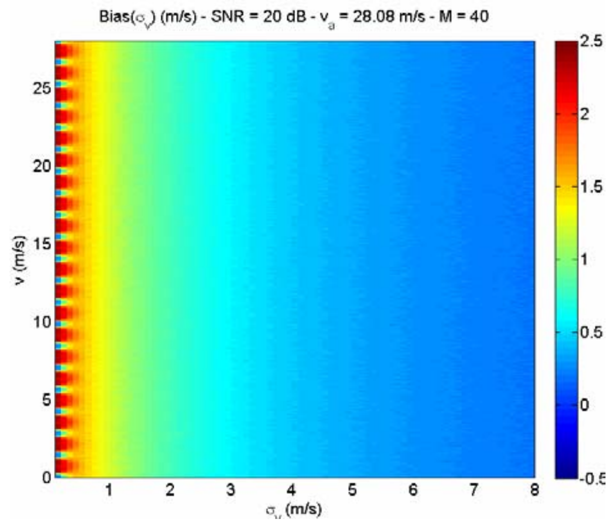

Fig. 3. ORDA spectrum width bias as a function of the true spectrum width and the mean Doppler velocity of the weather signal for data processed with the rectangular window. The SNR is 20 dB, T_S = 1 ms, and there are 40 samples in the time-series data.

4. PROPOSED SPECTRUM WIDTH ESTIMATOR

Modifying the estimator in (8) is straightforward by comparing it with the one in (6). An unbiased DFT-based autocorrelation estimator can be obtained from

$$\hat{R}_{UDFT}(l)=\frac{\hat{R}_{DFT}(l)-\dfrac{1}{M}\displaystyle\sum_{m=M-l}^{M-1}V^*(m)V(m+l-M)}{\dfrac{1}{M}\displaystyle\sum_{m=0}^{M-l-1}d(m)d(m+l)}\,,\quad(11)$$

where the subscript UDFT stands for unbiased DFT-based estimator.

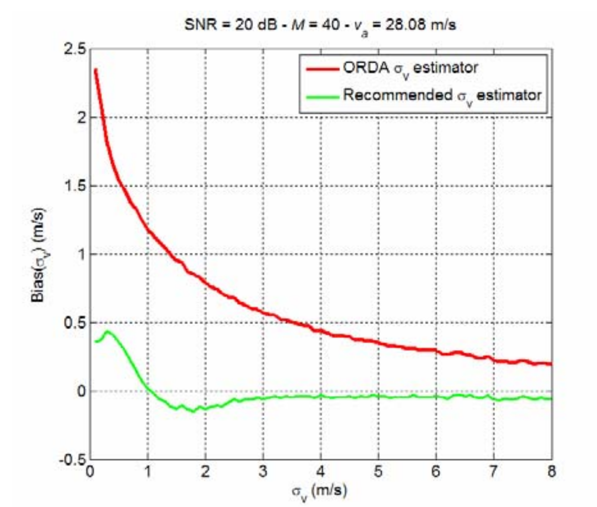

Fig. 4. Bias of the ORDA and the recommended spectrum width estimators as a function of the true weather signal spectrum width for data processed with a rectangular window.

With these modifications, the autocorrelation estimator in (11) is mathematically equivalent to the one in (6). This estimator is unbiased, does not include the spurious non-coherent terms, and naturally handles data windowing effects without the need of empirical adjustments.

Unfortunately, access to the time-series data is needed in order to implement (11), and this becomes problematic after any kind of spectral processing (e.g., a spectral clutter filter). In such cases, the only way to remove the spurious terms from the circular convolution is to take an inverse DFT and go back to the time domain before estimating the autocorrelation function. However, this would lead to a significant increase in computational complexity. Conveniently, spectral processing is usually preceded by tapered data windowing, and the contribution from the spurious terms is negligible as predicted by (10). Therefore, if a tapered window is applied, the following simplified autocorrelation estimator can be implemented:

$$\hat{R}_{SUDFT}(l) = \frac{\hat{R}_{DFT}(l)}{\frac{1}{M}\sum_{m=0}^{M-l-1} d(m)d(m+l)}, \qquad (12)$$

where the subscript SUDFT stands for simplified unbiased DFT-based estimator.

Simulations were carried out to compare the performance of the unbiased DFT-based estimator to the classical pulse-pair estimator. Fig. 4 shows the bias of both estimators as a function of the true spectrum width of the weather signal. Although not completely unbiased, the recommended spectrum width estimator exhibits a much smaller bias than the current ORDA spectrum width estimator. This bias is the largest for narrow spectrum widths, and is the same as the one observed in the legacy RDA system.

5. CONCLUSIONS

It was shown that the autocorrelation estimator in ORDA is biased, and this problem is exacerbated if using a rectangular data window. If using tapered data windows, the bias is corrected with empirical adjustment factors; however, if using a rectangular data window, the spectrum width estimate is not adjusted and the biased nature of the autocorrelation estimate translates into a significant bias in the spectrum width estimator. As a result, ORDA cannot use a rectangular window to process time-series data without compromising the performance of operational algorithms that rely on the spectrum width (e.g., the radar echo classifier, or the turbulence detection algorithm).

An unbiased DFT-based autocorrelation estimator was recommended for implementation in the ORDA subsystem. This estimator is mathematically equivalent to the classical pulse-pair estimator implemented in the legacy RDA as it eliminates the spurious terms and handles spectrum broadening due to data windowing analytically.

Currently, ORDA operates with the Hamming window by default. By utilizing an unbiased autocorrelation estimator, ORDA will be able to operate with the rectangular data window and realize a 30% reduction in spectral moment estimate errors.

This recommended change to the ORDA spectrum width estimation is not a comprehensive solution. The estimator in (2) has important limitations that prevent efficient operational use of the spectrum width. The recommended modifications presented here and approved by the NEXRAD Technical Advisory Committee earlier this year are just a short-term solution to allow the use of the rectangular window and produce more accurate spectral moment estimates. In fact, improved spectrum width estimators are currently being evaluated for their future implementation in the ORDA (Meymaris and Williams 2007). With upcoming ORDA upgrades, the NEXRAD network will be able to produce better spectrum width estimates which will result in improved warnings of severe weather and increased accuracy in detecting hazards to aviation.

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